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CONVERGENCE AND GROWTH  
AMONGST RICH AND POOR

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I am grateful to Shahrukh Khan for stimulating my interest in these issues and for many helpful discussions. My thanks to David Kennett for his comments on an earlier version. The usual caveat applies.

# CONVERGENCE AND GROWTH AMONGST RICH AND POOR

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Abstract

This paper reexamines the Baumol-Wolff (1988) hypothesis that there exists an exclusive international Convergence Club of rich nations whose members will move nearer each other with the passage of time. We argue that the Baumol-Wolff analysis does not really justify such a claim except in a relatively weak sense. Even in this weak sense we find that it is the poorer nations and not the richer who show convergent tendencies. We also show that the Diffusion hypothesis of Gomulka (1971) offers better hopes of identifying such a Convergence Club in a more meaningful sense. Using a generalisation of the Gomulka hypothesis, we find that there are two mutually exclusive Convergence Clubs - one for the richer nations and one for the poorer ones.

JEL Classification : O3, O4

Key Words : Convergence Club, Growth

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## CONVERGENCE AND GROWTH AMONGST RICH AND POOR

The behaviour of economies in the long run has always been close to the centre of economic analysis. The process of long run growth was a major concern of the founding fathers of the subject - Smith, Ricardo and Marx amongst others. After a relatively dormant period, interest in long run growth economics revived after World War II in response to the pressing need for reconstruction in Europe and the gradual emergence of the ex-colonial nations.

In the generation or so since the countries of Asia and Africa became independent <sup>1</sup>, there have been some outstanding success stories - but on the whole the record is depressingly grey. In many countries growth has been low and patchy. The disparity between the rich and poor nations remains very wide. A natural question which then arises is whether the current situation is a cyclical phenomena or part of a longer term trend. Put differently, an important issue is the question of whether or not nations tend to approach each other or move away from each other - the so called "convergence" issue. If indeed the poor nations of today are eventually going to converge to the rich nations, then one may regard the plight of the poor nations as a temporary phase: an adjustment period during which they will catch up. Convergence suggests the relative unimportance of initial conditions to prosperity in the long run. It may also be interpreted as suggesting that in the growth process there are eventually disadvantages to an early start - the rich do grow more slowly than the poor.

Like other "grand" themes that have percolated in economics, convergence needs to be defined more succinctly and unambiguously if it is to have much operational value. <sup>2</sup> The purpose of this study is, in part, to examine a number of alternative interpretations of convergence and to use these to compare previous empirical studies which have a bearing on the convergence issue. Another purpose of the study is to provide a more general approach which will enable empirical investigation of the convergence issue.

In recent years the issue of convergence has been examined extensively by Baumol (1986), Baumol and Wolff (1988) and Baumol, Blackman and Wolff (1989). The essential message of these studies is that convergence, or catching up, is occurring at the international level but only for a subset of countries, whilst the bulk of countries are excluded from the "Convergence Club". Thus what drives growth for the members of the Convergence Club is their initial conditions. A separate but complementary view is that of Gomulka (1971, 1986) who argued that technology transfer via diffusion would allow catching up albeit country specific factors would play a part. He did not specifically address the Convergence Club issue. It is my contention that Gomulka's analysis is in fact an appropriate vehicle for the discussion of the Convergence Club issue. A common feature of both these analyses is that "initial conditions" are captured in a very simple way - essentially by real GDP per capita or real GDP per capita relative to the world leader. We retain this simple approach throughout the paper.<sup>3</sup>

The rest of this paper is organised as follows. In Section 1 we outline the approach of Baumol et al (hereafter referred to as the BW approach) and show that the existence of a small Convergence Club is dependent upon an unusual and somewhat narrow interpretation of convergence. A more conventional definition of convergence using steady state analysis would imply a large widening of the Convergence Club. Such an analysis is conducted in Section 2. In Section 3 we analyse the approach of Gomulka and show that his approach does imply (conventional) convergence and the existence of an exclusive Convergence Club for certain parameter values. In Section 4, we discuss the econometric models based on the earlier theoretical analysis and present the results whilst Section 5 concludes the paper.

## 1. The Exclusivity of the Convergence Club

The basis of the BW view that there exists an exclusive Convergence Club is a regression of the form :

$$\ln (Y_T / Y_0) = a + b Y_0 - c Y_0^2 \quad (1)$$

where  $Y$  stands for per capita real GDP <sup>4</sup> and the subscripts 0 and T stand for initial and terminal period respectively. We shall refer to the interval  $[0, T]$  as a "generation". Baumol and Wolff (1988) estimated (1) using the Heston-Summers data set for 72 countries with 1950 as the initial period and 1980 as the terminal period. The estimates they obtained were  $\hat{a} = 0.586$ ,  $\hat{b} = (38/10^5)$  and  $\hat{c} = (1/10^7)$  <sup>5</sup>.

For each country, the LHS of (1) is a measure of the growth rate of  $Y$  over the generation whilst the RHS is a quadratic in initial  $Y$ . The quadratic expression has a unique maximum at  $Y_0 = (b/2c) = \$ 1900$ . Clearly for countries with an initial real per capita income in excess of this critical value growth is inversely related to initial level.<sup>6</sup> Consider any two countries A and B in this set and suppose that initially A is richer than B. Then the ratio of the per capita real incomes of A to B will be lower at the end of the generation than at the beginning. BW call this set of countries the Convergence Club. The reverse is true for those countries whose initial  $Y$  is below the critical level. This is illustrated in Figure 2. <sup>7</sup>

Clearly, so long as the focus of attention is one generation only, there is some merit to defining the Convergence Club as the set of countries for whom growth and initial level are negatively correlated. However this condition of negative correlation between growth over the period and initial level is neither necessary nor sufficient for the variance of real per capita income to be lower at the end of the period than at the beginning. <sup>8</sup> Quite simply the absolute gap between two BW Convergence Club members can be bigger at the end of the generation than it was at the beginning. This can be demonstrated as follows. Suppose A and B belong to the BW

Convergence Club so that  $\frac{Y_T^A}{Y_T^B} < \frac{Y_0^A}{Y_0^B}$ . This carries no implication for the ratio of

the absolute value of income differences between A and B which is  $\frac{|Y_T^A - Y_T^B|}{|Y_0^A - Y_0^B|}$ .

More importantly the BW analysis offers no concrete answer to the question : *convergence to what ?* Suppose the same growth process implicit in (1) were to repeat

itself generation after generation. The question that naturally arises then is : *does there exist a steady state towards which some countries converge and what are the characteristics of this steady state ?*

Abramovitz(1985) has suggested that convergence should be interpreted as implying a long run tendency towards the equalisation of levels of per capita income or levels of per worker product. We retain this interpretation and define convergence as requiring two conditions. First, the existence of a steady state in which per capita real income is equalised; and secondly the presence of dynamic forces which in the long run drive the world economy to this steady state. The existence of an exclusive Convergence Club is then taken to imply the existence of a non-exhaustive set of countries which in the long run are driven to this steady state with equalised real per capita incomes.

We turn in Section 2 below to an analysis of such a possibility but staying within the BW paradigm.

## 2. Steady State Analysis and Convergence

For the purpose of analysing long run convergence, we recast (1) in a standard difference equation framework. Redefining  $Y_T$  as  $Y_t$  and  $Y_0$  as  $Y_{t-1}$  we can rewrite (1) as :

$$\ln Y_t = a + \ln Y_{t-1} + b Y_{t-1} - c Y_{t-1}^2 \quad (2)$$

The existence of a steady state equilibrium requires  $Y_t = Y_{t-1} = Y$  (say)

Hence the steady state level of real per capita income is given by :

$$cY^2 - bY - a = 0 \quad (3)$$

which implies a steady state equilibrium value for real per capita income given by :

$$Y^* = \frac{b + \sqrt{b^2 + 4ac}}{2c} \quad (4)$$

Given BW's estimates this turns out to be \$ 4977. <sup>9</sup>

To examine the issue of whether the world economy converges to this equilibrium we rewrite (2) using the transformation  $\ln Y_t = y_t$  or  $Y_t = e^{y_t}$ . Hence

$$y_t = a + y_{t-1} + b e^{y_{t-1}} - c e^{2y_{t-1}} = F(y_{t-1}) \quad (5)$$

Convergence to the steady state value of  $y^* = e^{Y^*}$  will occur if the absolute value of the slope of  $F$  at the equilibrium is less than unity. A convergent case with  $0 > F' > -1$  is shown in Figure 2.

The steady state equilibrium is where  $y = F(y)$  i.e. where the 45 degree line cuts  $F()$ .

Since the slope of  $F$  is given by  $F'(y) = 1 + b e^y - 2c e^{2y}$  and  $e^y = Y$ , we can calculate the slope of  $F$  at  $E$  as:

$$\rho = 1 + bY^* - 2c\{Y^*\}^2 \quad (6)$$

For BW's estimates, the calculated value of  $\rho$  is -2.06 which is greater than 1 in absolute value. Hence there is no convergence to the steady state. *The Convergence Club is empty!*

The analysis of convergence would be simpler if one slightly alters the fundamental BW model (equation (1)) to have the logarithm of initial per capita income as the RHS variable. This yields :



$$\ln (Y_T / Y_0) = a + b \ln Y_0 - c \ln Y_0^2 \quad (7)$$

which using our notation can be written as a simple difference equation in the logarithm of  $Y$  as :

$$y_t = a + (b + 1) y_{t-1} - c y_{t-1}^2 \quad (8)$$

The steady state equilibrium value is given by :

$$y^* = \frac{b + \sqrt{b^2 + 4ac}}{2c} \quad (9)$$

Here the negative root for  $y$  is discarded despite being a theoretical possibility since it implies a steady state equilibrium income which is a fraction.

The sufficient condition for convergence is that  $-1 < \rho < 1$  where  $\rho$  is given by the slope of  $F(y) = a + (b+1)y - cy^2$  at  $y^*$ . Thus

$$\rho = b + 1 - 2cy^*$$

Once again there is no reason to believe that an exclusive Convergence Club exists. Indeed given (7) and (8) the implication of  $\rho < 1$  (in absolute value) is that the variance of  $y$  is less at the end of a generation than at the beginning.<sup>10</sup> This implies a sort of convergence within a generation. Thus both within a generation for countries reasonably close to  $y^*$  and in the long run for all countries we get the same condition for convergence.

However even reinterpreting the BW model in this way we are left with a worry. In

long run equilibrium there is no growth. If the steady state equilibrium could be reformulated so as to include positive growth, this would imply the level of per capita income could not be tied down. Instead the ratio of per capita income to some other target could be tied down. This is precisely the approach taken by Gomulka to which we now turn.

### 3. Diffusion and Growth

The central conceptual apparatus derives from Gomulka(1971,1986). The basic idea is that the growth rate of technological change in any country depends on the technology gap between the country and the world leader in technology. The process whereby this happens is technology transfer and innovation. <sup>11</sup> This relationship is also hypothesised to be "hat" shaped or inverted U shaped. This is because countries with a very small gap have little pressure to imitate the leader whilst countries with a large gap have high pressure to mimic but lack the ability to do so. Hence for very different reasons both these groups have low technological growth. By contrast countries with a middle sized gap have enough pressure to mimic and also enough infrastructure, sufficient high quality education, a reasonably developed research and development sector etc to be able to exploit the gains from technology transfer. Empirical support is adduced using data from 55 countries for 1950-1968 (Gomulka,1971) and from the Eastern European countries (Gomulka,1986).

In operational terms, the growth rate of technology is proxied by the growth rate of real income per capita (or labour productivity) whilst the gap is proxied by the difference between the initial level of per capita income in the country to that of the world leader. In terms of our generational model we can express this as :

$$y_t - y_{t-1} = a + b (\bar{y}_{t-1} - y_{t-1}) - c (\bar{y}_{t-1} - y_{t-1})^2 \quad (10)$$

where  $\bar{y}_{t-1}$  is the initial log level of real income per capita in the leader country. Thus the gap is measured by  $\bar{y}_{t-1} - y_{t-1}$ . We have used the log level specification for the initial state because of its greater simplicity. To analyse the properties of this model, define  $z_t = \bar{y}_t - y_t$  as the gap.

For the leader the gap is always zero and hence from (10), it follows that :

$$\bar{y}_t - \bar{y}_{t-1} = a \quad (11)$$

This asserts that the growth rate of the leader country is exogenously given by  $a$ . Using (11) it follows that :

$$y_t - y_{t-1} = -z_t + z_{t-1} + a \quad (12)$$

From (10) and (12) we obtain the fundamental difference equation in the gap  $z$  as :

$$z_t = (1-b) z_{t-1} + c z_{t-1}^2 = F(z_{t-1}) \quad (13)$$

In long run equilibrium the gap is stationary so that  $z_t = z_{t-1} = z^*$ . Hence there are two potential equilibria :  $z_2^* = b/c$  and  $z_1^* = 0$ . To check the stability of each of the equilibria, note that  $F$  has a minimum at  $(b-1)/2c$ . Figure 3 illustrates for the case when  $0 < b < 1$  so that  $(b-1)/2c$  is negative.

In Figure 3,  $E_2$  is the unstable equilibrium since  $F' = (1-b) + 2cz = 1 + b > 1$  at  $E_2$ . At  $E_1$ , the value of  $F'$  is  $1 - b < 1$  and hence it is a stable equilibrium. Clearly countries with an initial gap in excess of  $b/c$  will not converge whilst others will. Hence there exists a well defined Convergence Club from which the poorest countries are excluded. All the countries in the Convergence Club converge to the per capita income of the leader and thereafter enjoy per capita income growth at the exogenously given rate  $a$ . Note that the model does not (and indeed cannot) specify the level of per capita income but instead specifies an equilibrium long run growth rate and an equilibrium *relative* per capita income.

#### 4. Econometric Specification and Results

We investigate the convergence issue by estimating slight modifications of the original BW and Gomulka specifications. In the case of BW, we use the double log formulation because of its greater analytical tractability; i.e we estimate (8). This formulation enables a test of convergence to a (zero growth) steady state as shown earlier. Gomulka's "gaps" model can be estimated in the form of (13) but this imposes rather than tests for convergence to a non-zero growth equilibrium. Hence we generalise the model slightly and estimate instead  $z_t = \phi(z_{t-1})$  where the functional form  $\phi$  is allowed to be determined by the data. We start by provisionally assuming that  $\phi$  has terms up to a quartic in  $z_{t-1}$ . We then use a variety of diagnostic and specification tests to search for a more parsimonious specification.

We use the updated Heston-Summers data set. For our initial period we used 1960 and for the terminal period 1985. This allowed us to include 109 countries in the sample including 17 "rich" countries. The tests were based on using real GDP per capita and the gaps were defined relative to the USA which was the leader both in 1960 and in 1985. Thus the definitions of the variables used in the estimation were :

$y_t$  = natural log of real GDP per capita in 1985

$y_{t-1}$  = natural log of real GDP per capita in 1960

$z_t = \bar{y}_t - y_t$

$z_{t-1} = \bar{y}_{t-1} - y_{t-1}$

where a bar on a variable denotes USA. Each of the vectors  $y_t$  through to  $z_{t-1}$  had 109 elements. Given the large sample size of 109, it was possible to appeal to asymptotic results when analysing the estimates. In using essentially cross section estimates to infer dynamic properties, we are in effect assuming parameter constancy over time.<sup>12</sup>

Table I below shows the results for the (modified) BW specification which is used to test for convergence in GDP per capita.

Table I : Convergence in GDP Per Capita

Eqn	const	$y_{t-1}$	$y_{t-1}^2$	$R^2$	FU( $\chi_1^2$ )	N ( $\chi_2^2$ )	H ( $\chi_1^2$ )
G	1.415 [2.73]	.657 [.747]	.029 [.050]	.799	5.53*	2.37	5.68*
L	-.158 [.334]	1.09 [.052]*		.80	.35	1.94	6.23*
Q	3.817 [.196]*		.074 [.003]*	.799	1.419	3.13	4.82*

Notes

(1) Dependent variable is  $y_t = \ln$  GDP per capita in 1985

(2)  $y_{t-1} = \ln$  GDP per capita in 1960

(3) figures in brackets are standard errors

(4) \* denotes significant at 5 %

(5) Sample size = 109

(6) The test statistics FU, N and H are the Reset test for Functional Form, the Jacques-Berra test for Normality of errors, and a test for Homoskedasticity against a simple heteroskedastic alternative. Under the null hypotheses of appropriate functional form, normality of errors and homoskedasticity, these are distributed as  $\chi^2$  with 1,2, and 1 degree of freedom respectively. Rejection of the null implies a misspecification.

The results are not highly satisfactory. The general model G, containing both linear and quadratic terms suffers from multicollinearity, but more importantly fails to pass either the functional form test or the simple homoskedasticity test. The implication is that G does not constitute an appropriate functional form. The special cases L(contains linear terms only) and Q(contains quadratic terms only) are clearly

superior. In both cases there is evidence of heteroskedasticity but the functional form appears adequate. Our preferred specification is Q because the evidence on heteroskedasticity is milder and because all the coefficients are significant at 5%.

A number of implications follow from the estimates of equation Q. Rewriting model Q as :

$$y_t = a + cy_{t-1}^2 = F(y_{t-1}) \quad (14)$$

it follows that no steady state exists because  $y = F(y)$  has no real solution. Defining  $g$  to be the growth rate, it follows from (14) that :

$g = \Delta y_t = a + (c-1)y_{t-1}^2$  has a minimum at  $y_{t-1} = (1/2c) = 6.76$ . This is illustrated in Figure 4 where the gap between  $F(y)$  and the 45% line measures growth over the "generation". It is clear that this gap is diminishing as the log of initial per capita income increases from zero to 6.76 but increasing thereafter. This estimate of the cutoff has an asymptotic standard error of .326 and hence is fairly robust.

Thus for all countries with an initial real per capita income of \$ 862 ( = exp 6.76) or less, growth is negatively related to initial level. But this weak convergence is not sufficient to ensure full convergence to a steady state equilibrium. With the passage of generations, growth will ensure that every country passes this cut off limit of \$ 862 and thereafter all countries will grow at rates positively related to their initial per capita incomes. Hence in time, large and ever widening disparities between nations will emerge. It should be noted that even this very limited kind of (intra generational) convergence is different in character from the BW results which suggested that the richer countries belonged to a (limited) Convergence Club.

Table II below shows the estimates for the generalised "gaps" model of Gomulka, viz :

$$z_t = \phi(z_{t-1}).$$

Table II : Convergence in "Gaps"

Eqn	const	$z_{t-1}$	$z_{t-1}^2$	$z_{t-1}^3$	$z_{t-1}^4$	$R^2$	FU ( $\chi_1^2$ )	N ( $\chi_2^2$ )	H ( $\chi_1^2$ )	LR ( $\chi_2^2$ )
G	.074 [.313]	.472 [1.00]	.243 [1.00]	.049 [.380]	-.022 [.050]	.81	.36	2.81	2.73	
Q		.518 [.176]*	.307 [.105]*		-.014 [.005]*	.81	.12	3.3	2.69	.23**
C		.387 [.224]*	.514 [.183]*	-.098 [.036]*		.81	.67	3.55	2.68	.67**

Notes

- (1) Dependent variable is  $z_t = [(\text{USA ln GDP per capita in 1985}) - (\text{ln GDP per capita in 1985})] = \text{"gap" in 1985}$
- (2)  $z_{t-1} = \text{"gap" in 1960}$
- (3) figures in brackets are standard errors
- (4) \* denotes significant at 5 %
- (5) \*\* denotes **insignificant** test statistic at 5%
- (6) Sample size = 109
- (7) The test statistics FU, N and H are the Reset test for Functional Form, the Jacques-Berra test for Normality of errors, and a test for Homoskedasticity against a simple heteroskedastic alternative. Under the null hypotheses of appropriate functional form, normality of errors and homoskedasticity, these are distributed as  $\chi^2$  with 1, 2, and 1 degree of freedom respectively. Rejection of the null implies a misspecification.
- (8) The test statistic LR is the Likelihood Ratio Test for Variable Deletion. Under the null hypothesis that the variables may be dropped from G, it has a  $\chi^2$  distribution with appropriate degrees of freedom. An insignificant value implies that the imposed variable deletions are acceptable.

The results are much more robust than those in Table I. The most general version of  $\phi$  we estimated included terms up to a quartic in  $z_{t-1}$ . This is model G in Table II. Apart from multicollinearity, the results are satisfactory. The functional form test, normality test, and the homoskedasticity test are all passed. The special cases Q (quartic) was obtained by simultaneously deleting the constant term and the cubic term from G whilst the other special case C (cubic) was obtained by simultaneously deleting the constant term and the quartic term from G. Both Q and C appear to be reasonable specifications. In both cases, the tests for functional form, normality and homoskedasticity are passed; the likelihood ratio test for the deletion of two variables is also passed; the goodness of fit is more than adequate for a cross section model. A marginal preference for Q over C is based on the fact that whilst in Q every coefficient is significant at 5%, in C all but the coefficient of  $z_{t-1}$  is significant at 5% whilst that coefficient is significant at 8.5%.

Despite this we base our further calculations on C rather than Q. This choice is based on the purpose behind the analysis. If a steady state exists, this implies that the equation  $z = \phi(z)$  has real roots, or equivalently that the fixed points of  $\phi$  are real. Stability analysis requires calculation of  $\phi'$  in the neighbourhood of the fixed points. Obtaining the fixed points and the derivatives of  $\phi$  and their asymptotic standard errors is obviously much easier if  $\phi$  is analytically tractable. It is much easier to solve these using C rather than Q. Also as we show later, the fixed points of Q and C are not distinguishable which further strengthens the case for using C. We turn to the analysis below.

Rewriting C as :

$$z_t = A_1 z_{t-1} + A_2 z_{t-1}^2 + A_3 z_{t-1}^3 = C(z_{t-1}) \quad (15)$$

the fixed points of  $C(z)$  can be calculated by solving  $z = C(z)$  to yield the steady state values as :



$$z_1 = 0 \quad (16a)$$

$$z_2 = \left[ -A_2 + \sqrt{A_2^2 - 4A_3(A_1 - 1)} \right] / 2A_3 \quad (16b)$$

$$z_3 = \left[ -A_2 - \sqrt{A_2^2 - 4A_3(A_1 - 1)} \right] / 2A_3 \quad (16c)$$

Using the above yields the following estimates and asymptotic standard errors :

Parameter	Estimate	Std Error
$z_1$	0	
$z_2$	1.828	.195
$z_3$	3.415	.240

The above calculations suggest that the parameters  $z_2$  and  $z_3$  are very well determined. These results are illustrated in Figure 5 in which the function  $C(z)$  is plotted against the 45% line. The values of  $C(z)$  are nothing more than the fitted values of the dependent variable using specification (C). The three equilibria are labelled  $E_1, E_2$  and  $E_3$  respectively. It is clear that the slope of  $C(z)$  is less than unity at  $E_1$  and  $E_3$  but greater than unity at  $E_2$ . Thus  $E_1$  and  $E_3$  are stable equilibria whilst  $E_2$  is unstable.<sup>13</sup> Hence those countries with an initial "gap" less than  $z_2 = 1.828$  converge to the high equilibrium of zero gap ; however those countries with an initial "gap" in excess of 1.828 converge to the "low level equilibrium trap " where the equilibrium gap is 3.415. Thus there are two mutually exclusive Convergence Clubs - one for the "rich" nations and one for the "poor" where the cutoff between rich and poor is an initial gap of 1.828. The mean gap in the sample in 1960 was 1.933 with a standard deviation of 0.859. Thus approximately 48% of the countries will converge to the leader whilst the remainder will converge to another equilibrium in which the relative GDP per capita will be very nearly 1/30 that of the USA<sup>14</sup> This does not imply absolute poverty since growth will be occurring and a real income of 1/30 of the USA may in fact be quite a handsome number.

All of the above analysis was based on the cubic specification C in Table II, despite our marginal preference for Q. As a final plausibility check, we graphed  $Q(z)$  against the 45% line in order to graphically calculate the fixed points of  $Q(z)$ .<sup>15</sup> The results are shown in Figure 6 : clearly the equilibria based on  $Q(z)$  are very close to those based on  $C(z)$  and the stability properties are the same. This is because the gap between  $C(z)$  and  $Q(z)$  is negligible. A regression of the fitted values using (Q) against those using (C) yielded a coefficient of .997 and a  $\bar{R}^2$  of .999. This confirms that qualitatively there is almost no difference between the two specifications and the greater analytical tractability of (C) justifies its use.

## 5. Conclusions

The issue of convergence and the existence of a Convergence Club is a complex one. The first formal attempt to define such an exclusive club was by Baumol(1986) and Baumol and Wolff (1988). Their analysis was somewhat limited by their model specification. We have shown that their approach can define an exclusive Convergence Club only in a very narrow sense - what we called weak convergence. This refers to a situation in which **within** one "generation" growth rates of GNP per capita are negatively related to the initial level of GDP per capita. Using this weak definition of Convergence, our results indicated that it was the poor not the rich nations who were converging - contrary to BW. However another approach - the Diffusion model of Gomulka (1971,1986) offers richer possibilities, although Gomulka himself did not examine the Convergence Club issue. By reformulating and generalising that analysis we are able to examine convergence in a more meaningful sense - viz. convergence **over** "generations". Our results suggest that there are two mutually exclusive Convergence Clubs - one for the "rich" and one for the "poor" where the division between rich and poor is endogenously determined by the model and the data. This implies that some countries are caught in a Leibenstein (1957) type low level equilibrium trap and a "big push" will be required to get them out of it. It is perhaps apposite that the caveat regarding parameter constancy be recalled here. A "big push" should change the forces propelling growth and hence change the parameter values underlying our analysis. To determine what these forces are, how they work and how they might be influenced by a big push is a challenging task. One possibility is by a richer specification of the initial conditions. We hope to address this in future work.

## NOTES

- <sup>1</sup> Most of the Latin American countries have been independent for much longer.
- <sup>2</sup> Other examples of "grand" themes which had to undergo the same process of circumscription are deindustrialisation, dualism etc.
- <sup>3</sup> It is of course possible to widen the net when one considers initial conditions. Infrastructure, physical and human capital etc. may all be important determinants of the growth path. An approach which attempts to test this is that of Dowrick and Gemmel(1991).
- <sup>4</sup> It is perfectly possible to base the analysis on some other appropriate variable such as labour productivity.
- <sup>5</sup> There is clearly a misprint in the paper where the value of  $\hat{c}$  is reported as  $(9.9 / 10^7)$  or  $(1/ 10^6)$ . This does not square with later calculations done by the authors.
- <sup>6</sup> \$ 1900 is only an estimate of this critical level. If the error process in (1) is normally distributed, then the estimator  $(b/2c)$  is a maximum likelihood estimator and as such a consistent estimator of the true unknown cut off level of real GDP per capita. BW do not report a standard error associated with the \$ 1900 estimate. It is possible to estimate the standard error of the estimate of \$ 1900 from the estimated covariance matrix of the coefficients of (1). Thus one can test the significance of the estimate. The mere fact that each of the coefficients in (1) was significantly different from zero is neither necessary nor sufficient to reject the null hypothesis that the true value of the cut off level is zero. Rejection of this hypothesis is of course necessary before one can presume the existence of a Convergence Club, even in the weak sense.
- <sup>7</sup> All figures are at the end of the paper.
- <sup>8</sup> This point is elaborated further when we discuss a slight variant of the BW approach.
- <sup>9</sup> The other root is discarded because it is negative.
- <sup>10</sup> The result follows from the fact that  $\text{Var}(y_t) = \rho^2 \text{Var}(y_{t-1})$ . We alluded to this earlier in footnote 5.
- <sup>11</sup> Baumol (1986) advances a not dissimilar argument although the formal embodiment of the argument is quite different.

<sup>12</sup> This assumption though heroic, is probably no more so than assuming that growth parameters are invariant over the countries - which is implicit in the cross section estimates of BW and Gomulka.

<sup>13</sup> These can be verified formally by evaluating the derivatives of  $C(z)$  in the neighbourhood of the three equilibria.

<sup>14</sup> This calculation is based on  $\exp(3.415) = 30.4$

<sup>15</sup> Given the quartic term in  $Q(z)$ , no analytical solution is possible.

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FIG 1 : The BW Convergence Club

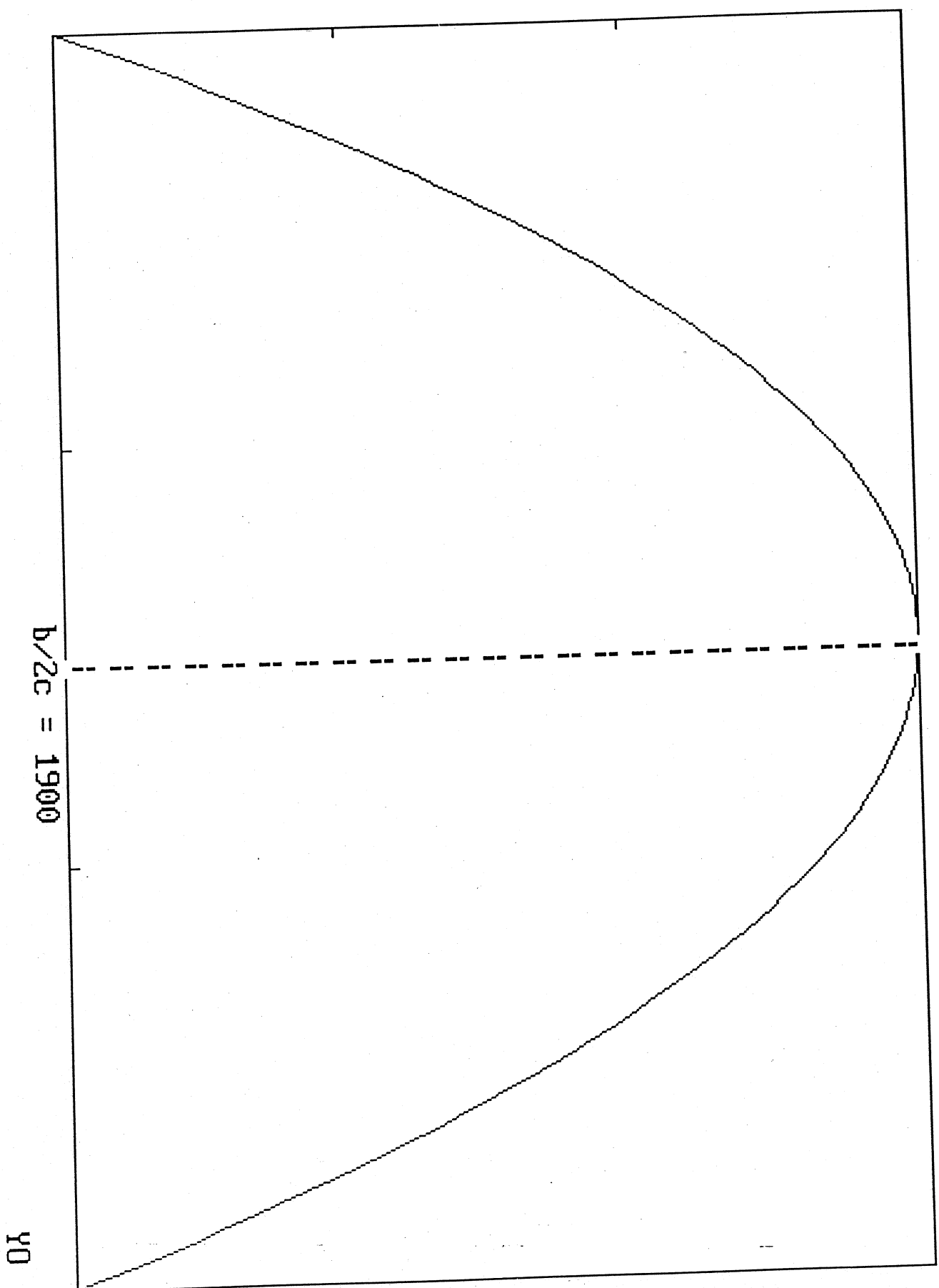


FIG 2 : Convergence to Steady State

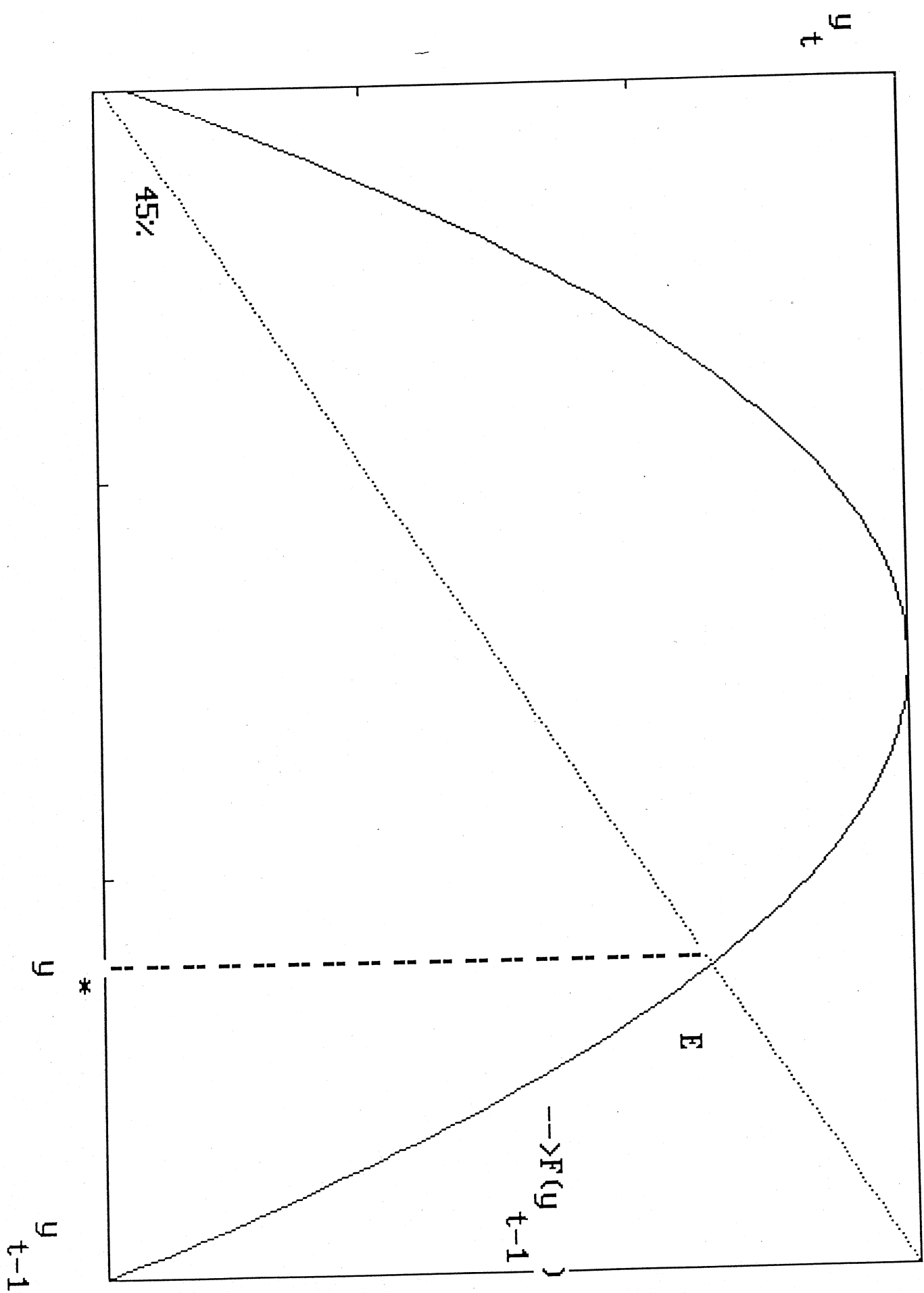


Fig 3: Convergence in Gaps

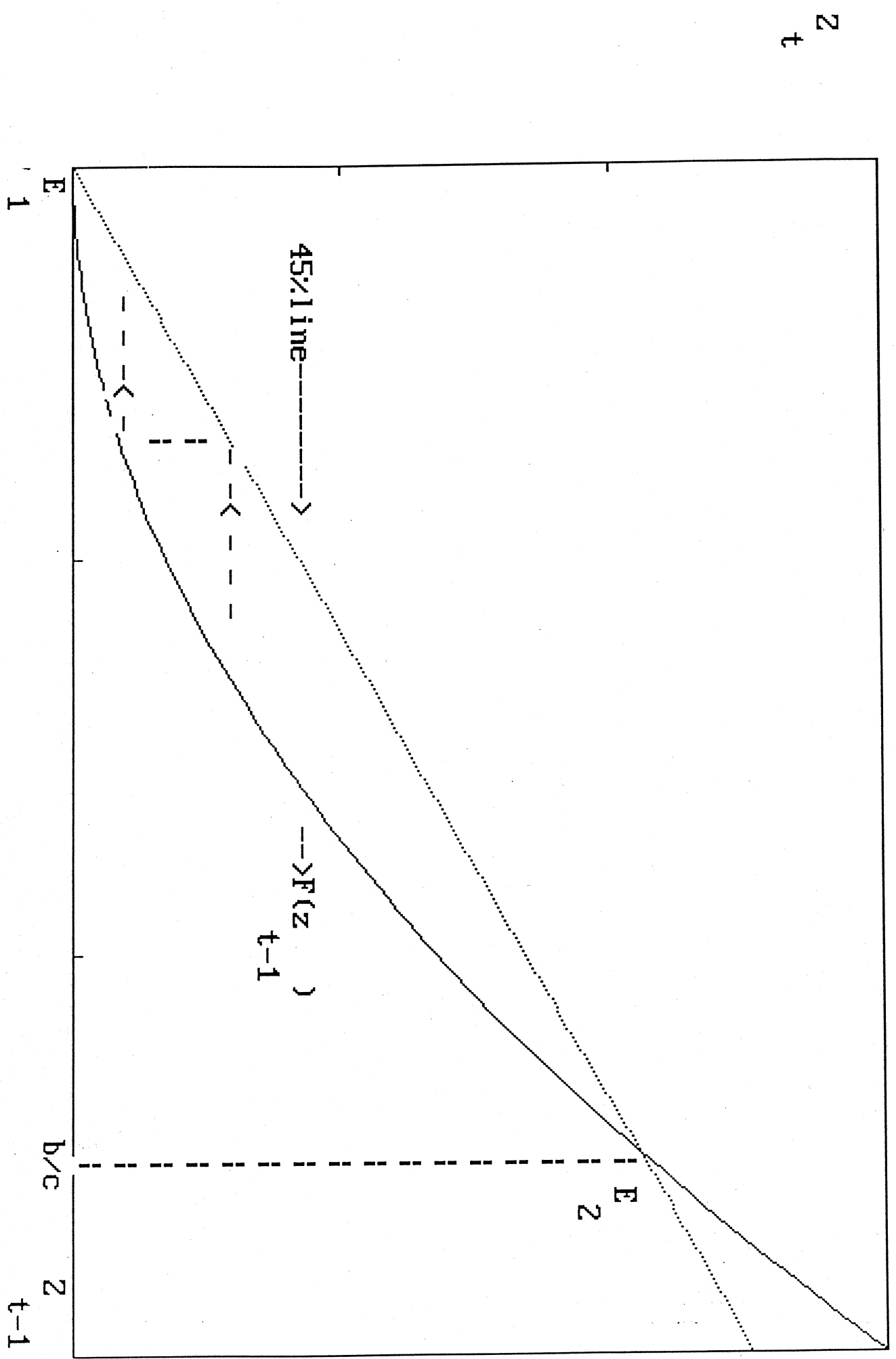




Fig 4: Convergence in GDP Per Capita

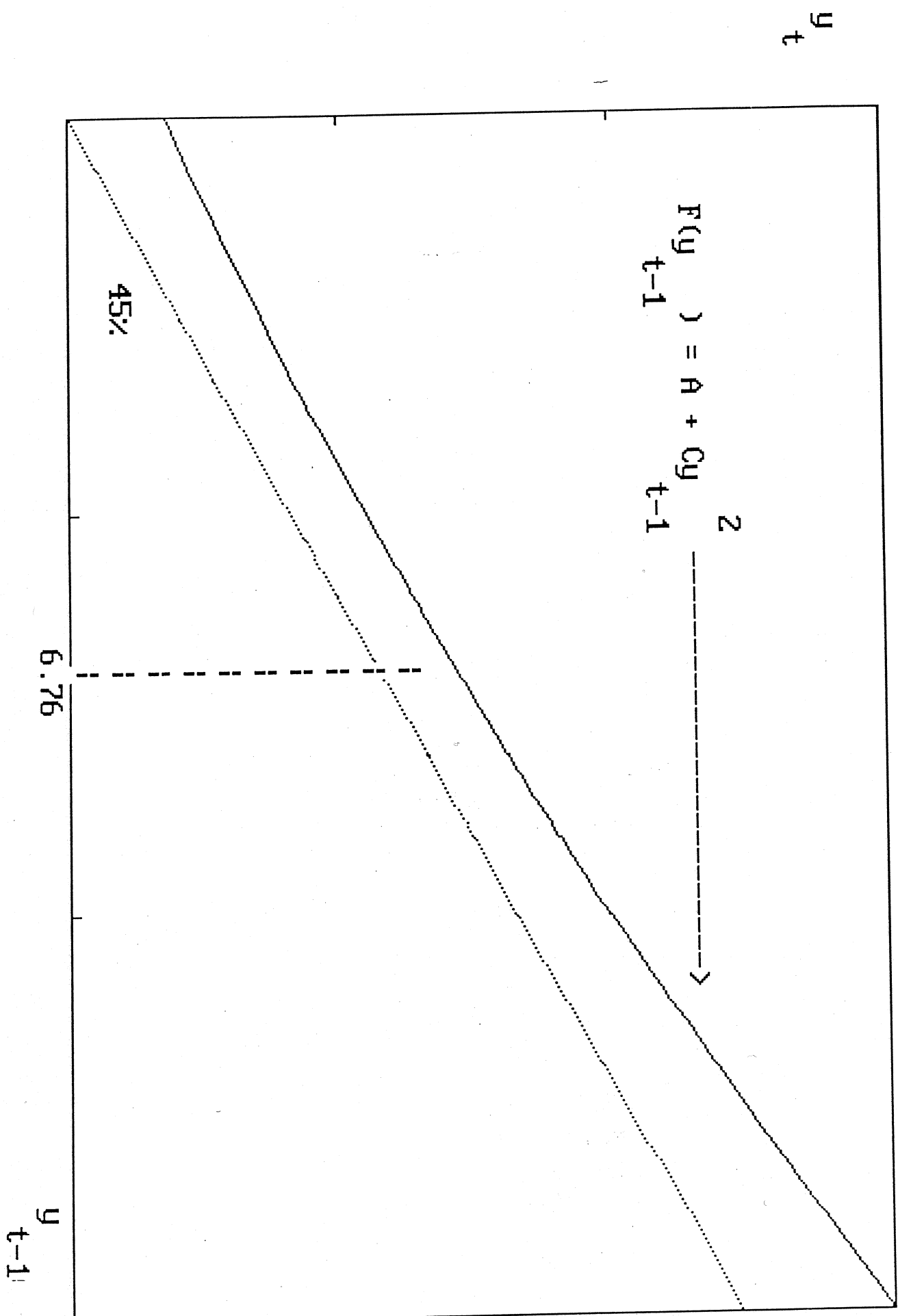


Fig 6: Gaps Equilibria with Quartic  $Q(z)$

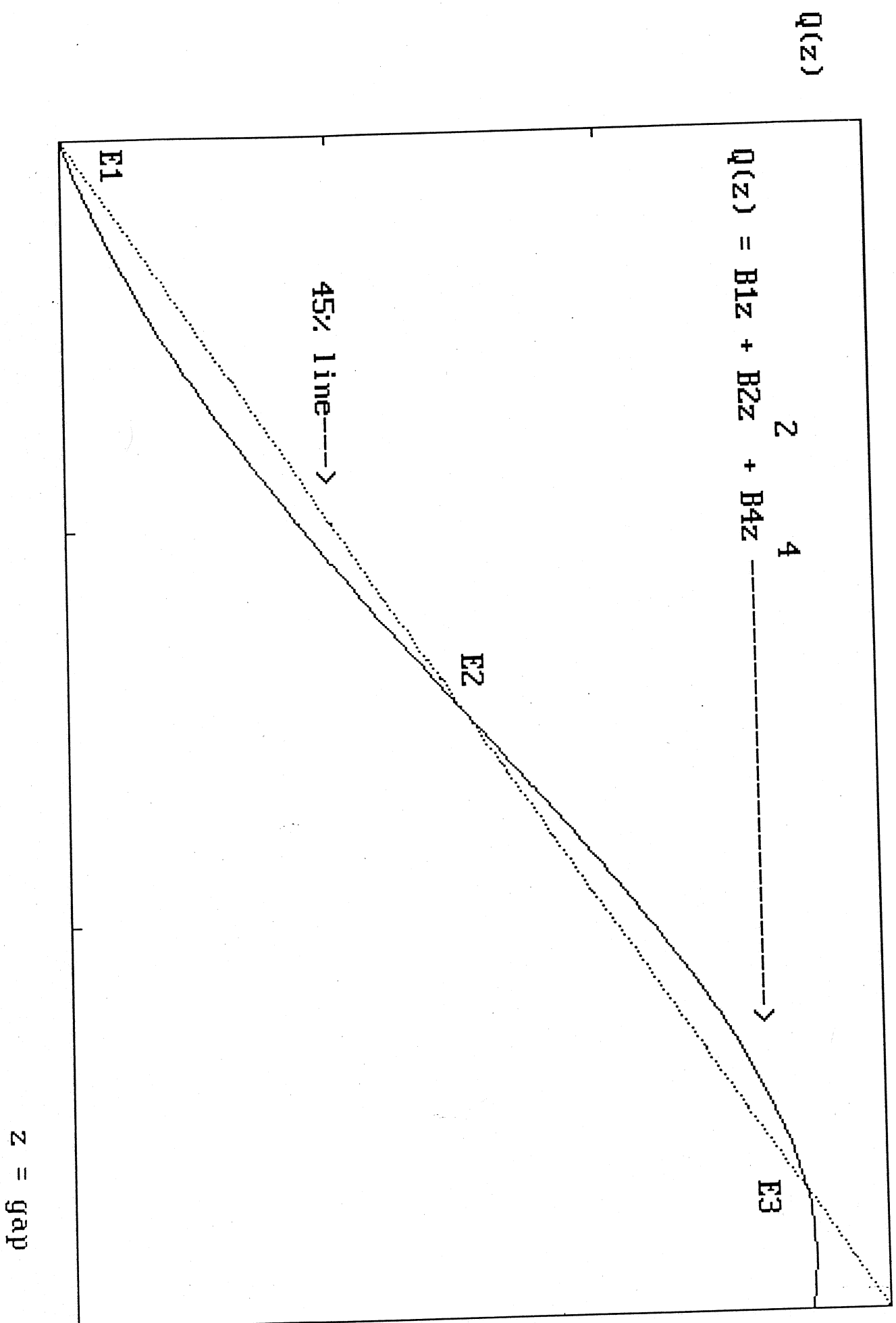


Fig 5: Existence and Stability of Equilibrium Gaps

